## 6.3 The CDF Technique

**Example 1:**

$$f(\mathrm{x}) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & o.w. \end{cases}$$

Let U=3X-1, find the pdf of U by the CDF technique.

**Example 2:**

$$f(x) = \begin{cases} \frac{x+1}{2} & -1 \le x \le 1 \\ 0 & o.w. \end{cases}$$

Let $U = X^2$ , find the pdf of U by the CDF technique.

**Example 3:**

$$f(\mathrm{x}) = \begin{cases} 2(1-x) & 0 \le x \le 1 \\ 0 & o.w. \end{cases}$$

(a) U=2X-1, find pdf of U
(b) U=-ln(X), find pdf of U
(c) Let $U = X^2$, find the pdf of U.

**Example 4:**

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 1 & 1 < x \le 1.5 \\ 0 & o.w. \end{cases}$$

Let U=10X-4, find the pdf of U by the CDF technique

**Example 5:**

$$f(\mathrm{x},\mathrm{y})=\begin{cases}3x & 0\le y\le x\le 1\\ 0 & o.w.\end{cases}$$

Let U=X-Y, find the pdf of U by the CDF technique.

**Example 6:**

$$f(\mathrm{x},\mathrm{y}) = \begin{cases} 1 & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0 & o.w. \end{cases}$$

Let U=X+Y, find the pdf of U by the CDF technique.

## 6.4. The Method of Transformation

**Example 1:**

$$f(\mathrm{x}) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & o.w. \end{cases}$$

Let U=3X-1, find the pdf of U by the transformation method.

**Example 2:**

Let $X \sim Beta\ (6,2)$, and $U=1-X$.

Find the pdf of U by the transformation method.

**Example 3:**

$$f(x) = \begin{cases} \frac{1}{4} & 1 \le x \le 5 \\ 0 & o.w. \end{cases}$$

Let $U = 2X^2 + 3$ , find the pdf of U by the transformation method.

**Example 4:**

Let X~ Exp (3), and $\mathrm{U} = \sqrt{X}$

Find the pdf of U by the transformation method.

## 6.5. The Method of MGF

**Example 1:**

Let $X_1, X_1, .....X_n$ be independently normally distributed with $E[X_i] = \mu_i$ and $V[X_i] = \sigma_i^2$ for i=1,2,...n

And let $a_1, a_2, .....a_n$ be constants. If $U = \sum_{i=1}^{n} a_i x_i$

Show that $U \sim N(\sum_{i=1}^{n} a_i \mu_i, \sum_{i=1}^{n} a_i^2 \sigma_i^2)$

**Example 2:**

Let $X_i \overset{iid}{\sim} \text{Gamma}(\alpha_i, \beta)$ for i=1,2,…n and $U = X_1 + X_2 + \ldots + X_n$

Show that $U \sim \text{Gamma}(\alpha_1 + \alpha_2 + \ldots + \alpha_n, \beta)$

**Example 3:**

Let $X_i \overset{iid}{\sim} \mathrm{Bin}(\mathrm{n}_i, \mathrm{p})$ , for i=1,2,...k and $U = X_1 + X_2 + ... + X_n$

Show that $U \sim \mathrm{Bin}(\mathrm{n}_1 + n_2 + ... + n_k, \mathrm{p})$

**Example 4:**

Let $X_1$ and $X_2$ be independent Poisson random variables with mean $\lambda_1$ and $\lambda_2$, respectively. Find

(a) The probability function of $U = X_1 + X_2$

(b) The conditional probability function of $X_1$ given that $X_1 + X_2 = m$

**Example 5:**

Let Z~N (0,1), and $\mathrm{U} = Z^2$

Show that $\mathrm{U} \sim \chi^2(1)$